$$\therefore \text{ the required average} = \frac{2v\int_0^\pi \sin\frac{1}{2}\theta d\theta}{\int_0^\pi d\theta} = 4v/\pi.$$

Also solved by *G. B. M. ZERR.*

MISCELLANEOUS.

56. Proposed by S. HART WRIGHT, A. M., M. D., Ph. D., Penn Yan, N. Y.

In latitude 40° N. $=\lambda$, when the moon's declination is 5° 23′ N. $=\delta$, and the sun's declination 9° 52′ S. $=-\delta'$, how long after sunset will the cusps of the moon's crescent set synchronously, the moon having recently passed its conjunction with the sun ?

[NOTE. Problem 56 is identical with problem 54, and need not be here reproduced. See January number, pages 27 and 28, for two solutions. EDIROR.]

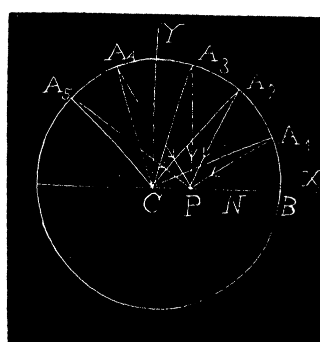
57. Proposed by GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

A particle is placed very near the center of a circle, round the circumference of which n equal repulsive forces are symmetrically arranged; each force varies inversely as the mth power of its distance from the particle. Show that the resultant force is approximately $\frac{m_1 n(m-1)}{2r^{m+1}} \times CP$, and tends to the center of the circle, where m_1 is the mass of the particle, CP its distance from the center of the circle, and r the radius of the circle.

I. Solution by the PROPOSER.

Let the particle be at P, and C the center of the circle. Suppose the forces to be at A_1, A_2, ... $CP=x$, and $\angle A_2CB=\theta$. Then $\angle A_1CA_2=\angle A_2CA_3 = \ldots . = 360°/n=\beta$, say. Draw A_2N at right angles to CB. Consider the force at A_2. Then,

$$X=[m_1/(A_2P)^m]\cos A_2PN=[m_1(r\cos\theta-x)] /(r^2+x^2-2rx\cos\theta)^{\frac{1}{2}(m+1)}.$$



Let $m_1/r^{\frac{1}{2}(m+1)}=M$. Then, since x is small, neglecting terms containing higher powers of x than the first, we have

$$X=M\left[r^{\frac{1}{2}(1-m)}\cos\theta+\frac{\cos 2\theta . x . r^{-\frac{1}{2}(m+1)}}{2} + \tfrac{1}{2}(m-1)xr^{-\frac{1}{2}(m+1)}\right].$$

To obtain the total force on P along X take the sum of n such expressions for all values of θ from $\theta=\alpha$ to $\theta=\alpha+(n-1)\beta$. Hence,

$$\Sigma X = M\left[\frac{r^{\frac{1}{2}(1+m)}\cos[\alpha+\frac{1}{2}(n-1)\beta]\sin\frac{1}{2}n\beta}{\sin\frac{1}{2}\beta}+\frac{\cos[2\alpha+(n-1)\beta]\sin n\beta}{2\sin\beta}xr^{-\frac{1}{2}(m+1)}\right.$$

$$\left.+n(m-1)xr^{-\frac{1}{2}(m+1)}\right]=\frac{m_1n(m-1)}{2r^{m+1}}x.$$

Similarly, for the force along Y,

$Y=[m_1/(A_2P)^m]\sin A_2PN=Mr\sin\theta(1-2x\cos\theta)^{-\frac{1}{2}(m+1)}=k\sin\theta+k_1x\sin2\theta$, where $k=Mr^{\frac{1}{2}(1-m)}$ and $k_1=M(m-1)r^{-\frac{1}{2}(m+1)}$.

For the sum of n such expressions, $\Sigma Y=0$. Hence, the resultant is

$$[(\Sigma X)^2+(\Sigma Y)^2]^{\frac{1}{2}}=\frac{m_1n(m-1)}{2r^{m+1}}\times CP.$$

Since the forces are equal and symmetrically arranged in the circumference, their resultant will act *towards* the center of the circle.

II. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.

It is assumed that the forces are placed at equal intervals along the circumference, that the repulsive action is proportional to the mass acted on, and that each force acts with unit intensity upon a unit mass at a unit's distance.

Because of the symmetrical arrangement the resultant force will act in the direction PC, C being the center of the circle and P the position of the particle whose mass is m_1.

Let the line connecting the forces with C make angles of α, 2α, 3α, etc., with the radius through P, so that $n\alpha=2\pi$.

The distance of the first force from P is $\sqrt{(r^2-2r.CP.\cos\alpha+CP^2)}$, the cosine of the angle which its direction makes with CP is

$$(r\cos\alpha-CP)/\sqrt{(r^2-2r.CP.\cos\alpha+CP^2)},$$

and the force itself is

$$m_1/(r^2-2r.CP.\cos\alpha+CP^2)^{\frac{1}{2}m}.$$

Writing d for CP, the component of this force along PC is

$$m_1(r\cos\alpha-d)/(r^2-2r\ d\ \cos\alpha+d^2)^{\frac{1}{2}(m+1)}.$$

Putting this in the form

$$m_1(r\cos\alpha-d)(r^2-2r\ d\ \cos\alpha+d^2)^{-\frac{1}{2}(m+1)},$$

expanding by the binomial theorem, multiplying, and neglecting the third and higher powers of d, this becomes,

$$m_1r^{-m-1}\{-d+[r-\tfrac{3}{2}(m+1)r^{-1}d^2]\cos\alpha+(m+1)d\cos^2\alpha$$
$$+\tfrac{1}{2}[m+1)(m+3)]r^{-1}d^2\cos^3\alpha\}.$$

The sum of all such components is

$$m_1 r^{-m-1}\{-nd+[r-\tfrac{3}{2}(m+1)r^{-1}d^2](\cos\alpha+\cos2\alpha+\cos3\alpha+\ \ldots\ldots\cos n\alpha)$$
$$+(m+1)d(\cos^2\alpha+\cos^2 2\alpha+\cos^2 3\alpha+\ldots\ldots\cos n\alpha)$$
$$+\tfrac{1}{2}[(m+1)(m+3)]r^{-1}d^2(\cos^3\alpha+\cos^3 2\alpha+\cos^3 3\alpha+\ldots\ldots\cos^3 n\alpha)\}.$$

By trigonometry,

$$\cos\alpha+\cos2\alpha+\ldots\ldots\cos n\alpha=\frac{\cos\frac{1}{2}(n+1)\alpha.\sin\frac{1}{2}n\alpha}{\sin\frac{1}{2}\alpha};$$

$$\cos^2\alpha+\cos^2 2\alpha+\ldots\ldots\ \cos^2 n\alpha=\tfrac{1}{2}\{n+\frac{\cos(n+1)\alpha.\sin n\alpha}{\sin\alpha}\};$$

$$\cos^3\alpha+\cos^3 2\alpha+\ldots\ldots\cos^3 n\alpha=\frac{\cos\frac{1}{2}(n+3)\alpha.\sin\frac{3}{2}n\alpha}{4\sin\frac{3}{2}\alpha}+\frac{3\cos\frac{1}{2}(n+1)\alpha.\sin\frac{1}{2}n\alpha}{4\sin\frac{1}{2}\alpha}.$$

The value of these series when $n\alpha=2\pi$ are 0, $n/2$, and 0, respectively.

Hence the expression for the approximate value of the resultant force reduces to

$$m_1 r^{-m-1}[-nd+(\tfrac{1}{2}n)(m+1)d],\ \text{or}\ \frac{m_1 n(m-1)}{2r^{m+1}}d.$$

Also solved by *G. B. M. ZERR*.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

95. Proposed by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military College, Washington, Miss.

Solve by arithmetic, if possible.

A man sold a house for $7500 and gained a certain per cent. on the cost. If the cost had been $16\frac{2}{3}\%$ less, his gain would have been 25% greater. Find the cost of the house.

96. Proposed by RAYMOND SMITH, Tiffin, Ohio.

How many acres in a square field whose diagonal is 10 rods longer than the side ?

*** Solutions of these problems should be sent to B. F. Finkel, not later than April 10.

ALGEBRA.

85. Proposed by J. M. COLAW, A. M., Monterey, Va.

Sum the infinite series

$$\frac{1}{1^2.2^2.3^2.4^2.5^2}+\frac{1}{2^2.3^2.4^2.5^2.6^2}+\frac{1}{3^2.4^2.5^2.6^2.7^2}+,\ \text{etc.}$$

86. Proposed by J. MARCUS BOORMAN, Consultative Mechanician, Counselor at Law, Inventor, Etc., Woodmere, Long Island, N. Y.

Solve $x^2+yz=16\ \ldots\ldots(A)$; $y^2+xz=17\ldots\ldots(B)$; $z^2+xy=22\ldots\ldots(C)$, for all the roots.

[This is Col. Titus' problem—see "Maseres' Tracts," pages 188-276—and is solved by Dr. Wallis in 51 pages, and by Mr. Frend in 38 pages, 8vo., but by the writer in 1 or 2 pages, 4to., or less. J. M. B.]